

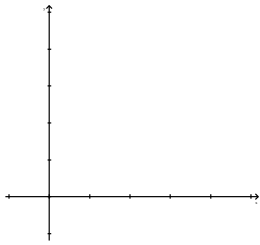
AP CALCULUS	LECTURE NOTES	MR. RECORD
Section Number: <b>3.2</b>	Topics: Rolle's Theorem and The Mean Value Theorem	Day: 1 of 1

## I. Rolle's Theorem

Activity:

Step 1: Place two points anywhere on the coordinate plane below that have the same y-values.

Step 2: Connect the two points with a continuous function that is also differentiable.



Conclusion: There MUST be at least one point on your function where you can draw a tangent line that is horizontal. (i.e. the slope is zero)

This activity basically models an important concept called **Rolle's Theorem**

### Rolle's Theorem

Let  $f$  be a continuous on the closed interval  $a, b$  and differentiable on the open interval  $a, b$ . If  $f(a) = f(b)$ , then there is at least one number  $c$  in  $a, b$  such that  $f'(c) = 0$ .

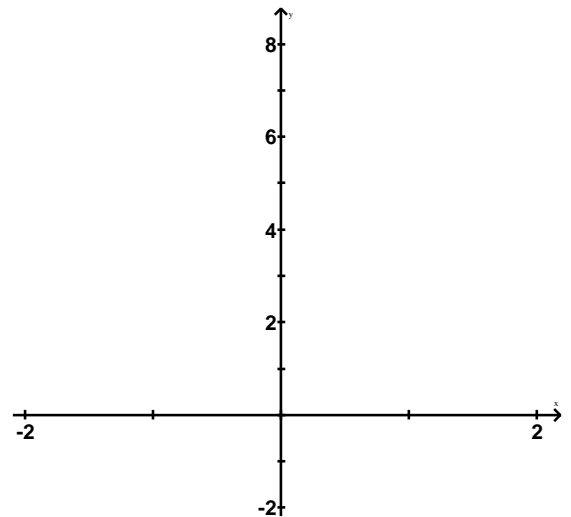
### Historical Note

#### ROLLE'S THEOREM

French mathematician Michel Rolle first published the theorem that bears his name in 1691. Before this time, however, Rolle was one of the most vocal critics of calculus, stating that the subject gave erroneous results and was based on unsound reasoning. Later in life, Rolle came to see the usefulness of calculus.

### Example 1: Illustrating Rolle's Theorem

Determine if Rolle's Theorem applies to  $f(x) = x^4 - 2x^2$  on the interval  $-2, 2$ . State thoroughly the reasons why or why not the theorem applies. If the theorem does apply, find the value of  $c$  guaranteed by the theorem.



## II. The Mean Value Theorem

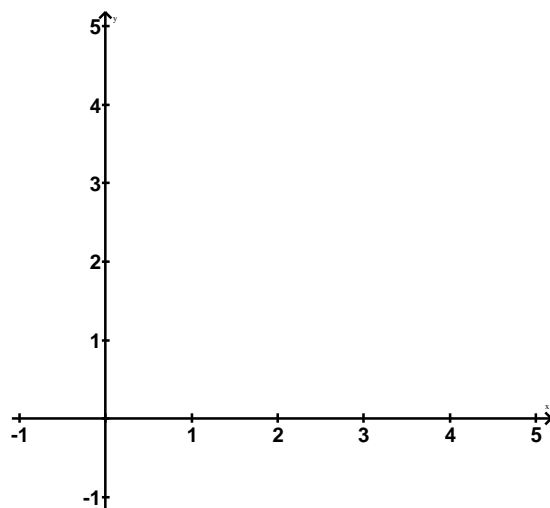
### The Mean Value Theorem

If  $f$  is continuous on the closed interval  $[a, b]$  and differentiable on the open interval  $(a, b)$ , then there exists a number  $c$  in  $(a, b)$  such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$

### Example 2: Illustrating The Mean Value Theorem

Determine if The Mean Value Theorem applies to  $f(x) = 5 - (4/x)$  on the interval  $[1, 4]$ . State thoroughly the reasons why or why not the theorem applies. If the theorem does apply, find the value of  $c$  guaranteed by the theorem.



### Example 3: Real World Application of The Mean Value Theorem

Two stationary police cars equipped with radar are 5 miles apart on a highway, as when in the figure. As a semi-truck passes the first patrol car, its speed is clocked at 55 miles per hour. Four minute later, when the truck passes the second patrol car, its speed is clocked at 50 miles per hour. Prove that the truck did or did not exceed the 55 mile per hour speed limit at some time between the two police cars.

